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AUTHOR Kocher, A. Thel
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ABSTRACT

The purpose of the present study was to investigate empirically the effects of violations of the assumption of homogeneity of within-group regression slopes on levels of significance in the F test of fixed-effects ANCOVA (analysis of covariance). The study used a Monte Carlo computer simulation procedure to generate data under the following conditions: (1) the covariate was a random variable; (2) the number of treatment groups was three; (3) treatment group n's were small to medium and were unequal as well as equal; (4) error variances were equal as well as unequal; and (5) covariate means were both equal and unequal. For each simulation, the proportion of F values exceeding certain critical values was compared to the nominal alpha corresponding to the critical values. (Author)

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AN INVESTIGATION OF THE EFFECTS OF NON-HOMOGENEOUS WITHIN-GROUP REGRESSION COEFFICIENTS UPON THE F TEST OF ANALYSIS OF COVARIANCE

A. Thel Kocher

Minneapolis Public Schools
Southeast Alternatives Project

Presented at the 1974 Annual Meeting of the American Educational Research Association, Chicago, Illinois.

Introduction

If a statistical technique can tolerate some degree of departure from its assumptions without seriously affecting the outcome of the experiment, the technique is considered to be "robust" with respect to those violations. Research has shown that the F test of analysis of variance (ANOVA) is very "robust" with respect to certain violations of its underlying assumptions. For a very comprehensive review of ANOVA "robustness" research see Glass, Peckham, and Sanders (1972). The remarkable "robustness" of the ANOVA F test leads, quite naturally, to speculation about whether the analysis of covariance (ANCOVA) F test would also show a high degree of "robustness" with respect to violations of its underlying assumptions.

Such speculation with respect to the consequences of violating the assumption of homogeneity of within-group regression coefficients in ANCOVA is not idle curiosity. The existence of non-homogeneous within group regression coefficients indicates that an interaction exists between the treatment variable and the control variable. When such an interaction exists, tests of the treatment effects by the F test of ANCOVA are not considered valid (Dayton, 1970; Feldt, 1958; Lindquist, 1953; Kempthorne, 1952). If the ANCOVA F test can be shown to be "robust" with respect to violations of the homogeneity of regression assumption; that is valid even when the homogeneity of regression assumption is violated, then experimenters will have at their disposal a second method--in addition to stratification--of capitalizing on a control variable.

Recent empirical investigations (Peckham, 1968; Robinson, 1969; McClaran, 1972) have provided some insight into the effects of violations of the assumption of homogeneity of within-group regression coefficients on levels of significance in the F test of ANCOVA. The results of the three studies have shown that when treatment group sample sizes are equal, the F test of ANCOVA is relatively "robust" to violations of the homogeneity of regression assumption. All three investigators concluded that for the majority of the differences that occurred the "actual" alpha values were less than the corresponding normal theory values. Only the McClaran (1972) study also considered cases in which the treatment groups had unequal sample sizes. The results did not indicate any definitive trend with respect to homogeneity of regression "robustness" when sample sizes were unequal and further research was recommended. All three of the studies considered the covariate values as being fixed over replications. All three also used data generation methods that resulted in unequal error variances from treatment population to treatment population thus violating a second assumption (homogeneity of variance) in addition to the one under primary investigation (homogeneity of regression). Because of these two factors Glass et al. (1972) have cautioned against overgeneralization of results.

Purpose

The purpose of the present study was to empirically investigate, under a variety of controlled conditions, some of the effects of violating the homogeneity of regression assumption underlying the F test of ANCOVA. Specifically, the study was concerned with

effects on levels of significance when the covariate was a random variable and the number of treatment populations was three. Parameters that were allowed to vary were: (a) treatment group sample sizes, (b) treatment population regression coefficients, and (c) treatment population covariate means. The treatment group sample sizes were allowed to vary such that cases of unequal as well as equal treatment group n's were investigated. The study considered situations in which the sample sizes, regression coefficients, and covariate means were combined in various ways with respect to size.

Procedure

The present study employed a Monte Carlo computer simulation technique to generate data meeting specified conditions. Two hundred forty-six different simulations were performed with each simulation consisting of 1800 relications of the ANCOVA F test on the data generated. When the usual model (Winer, 1971, p. 766) for ANCOVA is modified to allow for a unique regression coefficient β_j, in each treatment population, it yields

$$Y_{ij} = \mu_y + \tau_j + \beta_j (X_{ij} - \mu_x) + \varepsilon_{ij}. \qquad (1)$$

Since the present study assumed a true null hypothesis, the quantity $\mu_y + \tau_j$ was considered equal to a constant which was taken to be zero.

Thus, equation 1 becomes

$$Y_{ij} = \beta_j (X_{ij} - \mu_x) + \varepsilon_{ij} \qquad (2)$$

which was the basic form used to generate all data in the present investigation.

The investigation was conducted in two parts. In Part I, the means of the concomitant variable were equal in all three treatment populations. These situations are of the true experimental type with the covariate being used to increase the precision of the experiment. Part II considered instances in which the means of the concomitant variable differed from treatment population to treatment population. These situations are of a quasi-experimental nature and the covariate is primarily being utilized to reduce initial treatment group bias rather than to increase precision.

Once values were specified for the population covariate means, μ_{X_j}, and the population regression coefficients, β_j, Y_{ij} values were determined by generating values for X_{ij} and ϵ_{ij}. Each X_{ij} value was a "pseudo-random" number which can be thought of as having been drawn at random from a normally distributed population with mean of zero and variance of one, $N[0,1]$. The random number generator used to generate the X_{ij} values utilizes Marsaglia's second method (Marsaglia & Bray, 1964) to generate normal random deviates. The final step in obtaining the Y_{ij} value was to determine a value for ϵ_{ij}. The method of accomplishing this depended upon whether Case 1 or Case 2 was under investigation.

Case 1. In Case 1, ϵ_{ij} was thought of as representing a normal random deviate from a population with zero mean and variance equal to the variance error of estimate, $1 - r^2_{xy_j}$. Each ϵ_{ij} was obtained by using the random number generator to obtain a normal

random deviate and then multiplying the deviate by $\sqrt{1 - r^2_{xy_j}}$.

Thus, each value of the dependent variable is given by the function

$$Y_{ij} = \beta_j (X_{ij} - \mu_X) + \sqrt{1 - r^2_{xy_j}}\, z_{ij}, \qquad (3)$$

where z_{ij} represents the normal random deviate obtained from the random number generator. Since σ_{x_j} and σ_{y_j} were both equal to one, $r_{xy_j} = \beta_j$ and equation 3 may be rewritten as

$$Y_{ij} = \beta_j (X_{ij} - \mu_X) + \sqrt{1 - \beta_j^2}\, z_{ij} \qquad (4)$$

which was the form used to generate the data under the conditions of Case 1.

In addition to the homogeneity of regression assumption, the model for the fixed-effects ANCOVA requires that the error variances for all treatment populations be equal (homogeneity of variance). Since β_j varied from treatment population to treatment population, the quantity $1 - \beta_j^2$ which represents the treatment population error variance, was unequal from group to group which violated the homogeneity of variance assumption. As a consequence the variances of the adjusted criterion scores will also be unequal from treatment population to treatment population. Atiqullah (1964) and Potthoff (1965) have indicated that in the cases of nonnormality and heterogeneous variances respectively it is the characteristics of the covariate distribution rather than the characteristics of the criterion or error distributions which seem to affect "robustness." If this holds true in the case of nonhomogeneous regressions, then the unequal error variances will

not seriously contribute to lack of "robustness" since the covariate distributions all have equal variances.

In an attempt to determine whether or not unequal error variances seriously affect the "robustness" of the F test of ANCOVA and to gain some insight into the "interaction" of heterogeneous variances and regressions, the present study considered one other case.

Case 2. In Case 2 the function

$$Y_{ij} = \beta_j (X_{ij} - \mu_X) + Z_{ij} \qquad (5)$$

was used to generate the data. Since the error term in the ANCOVA model is a residual about the within-groups regression line, the variance of the error term for any treatment population is given by

$$\sigma^2_{\varepsilon_j} = \sigma^2_{Y_j} (1 - r^2_{XY_j}). \qquad (6)$$

It can be proven that the use of equation 5 to obtain pairs of criterion and covariate values yields data such that the error variances, $\sigma^2_{\varepsilon_j}$, as given by 6 are equal to one in all three treatment populations as are the $\sigma^2_{X_j}$ terms. In this case the equal covariate and error variance terms result in criterion score variances, $\sigma^2_{Y_j}$, which are unequal from treatment population to treatment population as are the variances of the adjusted criterion scores.

Results

The results of the present investigation into the effects of heterogeneous within-group regression slopes on Type I errors in the F test of ANCOVA are contained in Tables 1 and 2. Table 1 shows the results of simulations of the true experimental type in which the three covariate population means were equal (Part I). Table 2 contains the results of simulations of a quasi-experimental nature in which the three covariate population means were unequal (Part II). Each table shows, for nominal levels of significance .10, .05, and .01, the actual significance level where actual significance level is defined as the proportion of obtained F's greater than or equal to the appropriate critical value.

As suggested by Glass et al. (1972) interpretations of differences between actual and nominal significance levels should be based upon confidence intervals determined by calculating the appropriate standard errors of a proportion. For convenience, those actual values which were found to be significantly different from nominal when a 95% confidence interval was used have been marked with an asterisk.

Examination of the data for all simulations in which the homogeneity of regression assumption was not violated reveals that only one actual alpha value was significantly different from nominal. Additionally, the instances of actual values exceeding nominal and nominal values exceeding actual were approximately evenly distributed. These results indicate that the simulation process performed accurately.

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Insert Tables 1 and 2 about here

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Discussion

Since ANCOVA may be applied in situations where the treatment populations have either equal or unequal covariate means, the present investigation considered situations in which the covariate means were equal (Part I) and in which the covariate means were unequal (Part II). In an attempt to gain some insight into the "interaction effect" of heterogeneous error variances and regressions, the study was also designed to generate the data under two different error variance conditions — once with heterogeneous error variances (Case 1) and once with homogeneous error variances (Case 2). It should be noted that the variances of the adjusted criterion scores are heterogeneous under either the Case 1 or Case 2 method.

Combining the two covariate mean conditions and the two error variance methods allowed the data for the present investigation to be generated under one of four possible procedures: (a) covariate means equal and error variances heterogeneous (Part I, Case 1), (b) covariate means equal and error variances homogeneous (Part I, Case 2), (c) covariate means unequal and error variances heterogeneous (Part II, Case 1), and (d) covariate means unequal and error variances homogeneous (Part II, Case 2). Because the effects of violating the homogeneity of regression assumption appear to differ depending upon which of the four procedures was used, the results are discussed separately for each procedures.

Part I, Case 1

The results of the present study indicate that, under the

conditions of Part I, Case 1 — covariate means equal and error variances heterogeneous — the F test of fixed-effects ANCOVA is robust with respect to violations of the homogeneity of regression assumption when the treatment groups have equal sample sizes. The only significant discrepancies between actual and nominal alpha values occur for regression coefficients as disparate as .1, .5, and .9. While the trend is not entirely consistent, the actual alpha values tend to be slightly larger than nominal. The results of this portion of the present study coincide with the conclusions stated by Peckhan (1968) and McClaran (1972) with respect to the degree of heterogeneity of regression required before differences between actual and nominal alpha values become significant. However, in contrast to the results of the present study, both Peckham (1968) and McClaran (1972) found that the actual alpha values tended to be less than the corresponding nominal values. One possible explanation for the discrepancy is that both Peckham (1968) and McClaran (1972) considered the covariate as fixed over replications whereas the present study allowed the covariate to be a random variable.

With unequal rather than equal sample sizes, the F test is found to exhibit a definite lack of robustness. When the large regression coefficient is paired with the small n, a majority of the acutal alpha values are significantly different from nominal. Even with regression coefficients as similar as .4, .5, and .6 there are serious discrepancies between the actual and theoretical alpha values. In every situation, the actual value is less than nominal indicating that the F test of ANCOVA is actually more conservative under the specified conditions. When the large coefficient is paired with the large n, the lack of robustness is even more severe; with this pairing of regression coefficient and n, the actual alpha values are all greater than nominal. This indi-

cates that an experimenter using ANCOVA under the specified conditions would actually have more of a chance of making a Type I error than indicated by the nominal alpha level he has chosen.

Part I, Case 2

The results of the present study indicate that, as far as level of significance is concerned, the F test of fixed-effects ANCOVA is robust with respect to violations of the homogeneity of regression assumption when the data is generated by the method of Part I, Case 2 — covariate means equal and error variances homogeneous. Even with regression coefficients as different as .1, .5, and .9 the instances of significant differences between the actual and nominal alpha values are slight. The robustness appears to hold regardless of sample size combinations, theoretical alpha level, or regression coefficient and sample size combinations. Additionally, there does not appear to be any consistent trend for the actual alpha value to be larger or smaller than the nominal value in any situation.

Part II, Case 1

When the data were generated under the conditions of Part II, Case 1 — covariate means unequal and error variances heterogeneous — the results do not establish a clear trend with respect to the robustness of the ANCOVA F test when the homogeneity of regression assumtion is violated. When the sample sizes are equal the actual alpha values are significantly different from nominal only for regression coefficients as disparate as .2, .5, and .8. Theoretical alpha level does not seem to affect the degree of

robustness of the F test, nor does it matter whether the small mean is paired with the small or large regression coefficient. There also seems to be no effect whether n is 5, 10, or 20. All of the significant differences occur when the actual alpha value is greater than nominal, and the majority of the nonsignificant values also follow this trend.

When the treatment groups have unequal n's and when the n's and coefficients are paired smallest with largest, the F test of ANCOVA appears more robust than when the n's are equal. The effects of violations of the assumption appear more serious as nominal alpha increases as evidenced by the fact that most of the significant differences between actual and nominal alpha occur for a nominal alpha of .10. In most cases the actual alpha value is less than nominal; however, a notable exception occurs for n's of 10, 20, and 30 since many of the actual values are greater than nominal. As was the case with equal n's, whether the small or large covariate mean is paired with the small or large sample size and small or large regression coefficient does not appear to affect the results.

When the sample sizes are unequal and the coefficients and n's are paried smallest with smallest, the F test of ANCOVA exhibits an extreme lack of robustness with nearly all of the actual to nominal comparisons showing significance. Even with the regression coefficient set (.4, .5, .6) more than half of the actual to nominal comparisons are significant.

Part II, Case 2

Under the conditions of Part II, Case 2 — unequal covariate means and error variances homogeneous — the F test of ANCOVA appears to be moderately robust. When the sample sizes are equal in all three treatment groups, significant differences between actual and nominal values occur consistently only for simulations in which the regression coefficients are as disparate as .1, .5, and .9. There are a few other cases of significant differences especially as n becomes larger. The trend is for the actual alpha values to exceed the corresponding nominal values. The robustness of the F test does not appear to depend upon whether the small covariate mean is paired with the small or large regression coefficient nor does it appear dependent upon the theoretical alpha level being considered.

With unequal n's and the large coefficient paired with the small n, slightly more than half of the situations considered resulted in significant differences between actual and nominal values. Nearly all of the situations in which the extreme regression coefficients are as different as .2 and .8 show significant differences with some significant differences occurring for the coefficient set (.3, .5, .7). The trend is again for actual values to exceed nominal. The consequences of violating the homogeneity of regression assumption seem to be more serious at nominal alpha levels of .05 and .10 than at .01. The instances of significant differences also seem to depend somewhat on sample size.

When sample sizes are unequal and the small n is paired with

the small coefficient, the results are nearly identical to the results obtained when small n is paired with large coefficient.

General Observations

In a number of situations it has been noted in the present study that the lack of robustness seems to be more severe as sample size increases. The only situations in which this trend does not appear evident are those in which there are very few significant differences between actual and nominal alpha values or in which nearly all of the differences are significant. In each of these situations it seems highly unlikely that a trend could be established. In the first there are not enough significant differences to establish a trend and, in the other, no trend can be established because nearly all are significant.

Comparisons of the results from Case 1 and 2 under the conditions of Part I indicate that there are differences resulting from the manner in which the simulation data are generated. When the data are generated such that the error variances are unequal (Case 1), there are approximately five times as many significant differences between actual and nominal significance levels as there are when the data are generated so that the error variances are equal (Case 2).

Additionally, it can be seen that for almost all simulations, the results under the Case 2 conditions show actual values closer to nominal than when the data was generated under the procedures of Case 1. It appears, then, that the unequal error variances of Case 1 confound the effects of the heterogeneous regression slopes

and are primarily responsible for the significant differences that occur under Case I.

When the actual σ's obtained under the generation procedures of Case 1 are examined, it can be noted that the results follow the pattern that has been found to occur as a result of heterogeneous error variances (Glass et al., 1972). This seems to lend additional support to the author's belief that the differences which occurred when the data were generated under the conditions of Case 1 are due to the heterogeneous error variances or a combination of these and heterogeneous regression slopes rather than to the effects of the heterogeneous regression slopes alone.

Although the data from Part II of the present study does not point to as strong a confounding effect as did the data from Part I, there still seems to be ample evidence that such an effect also exists for Part II. Since a confounding effect does seem to exist for the results of both Parts I and II the author suggests that experimenters be guided by the results of Case 2 unless they have reason to suspect that they are using ANCOVA in a situation in which both regression slopes and error variances are unequal. It may well be that this is the usual situation; that is, it may well be that unequal error variances and heterogeneous regression coefficients tend to occur together and, thus, the results of Case 1 are most appropriate.

Summary and Recommendations

The present study investigated the effects, on levels of significance, of violating the assumption of homogeneity of regression

upon the F test of fixed-effects ANCOVA with the covariate a random variable. The study employed a Monte Carlo simulation technique to generate data meeting specified conditions. Two hundred forty-six different simulations consisting of 1800 replications of the ANCOVA F test on data meeting specified conditions. Each of the simulations involved three treatment populations. The parameters that were selected for each simulation were: (a) sample sizes, (b) regression coefficients, and (c) the covariate population means. The sample size combinations were selected from the following sets: (3, 5, 7), (5, 5, 5), (5, 10, 15), (10, 10, 10), (10, 20, 30), and (20, 20, 20). The regression coefficients were set at differing degrees of heterogeneity varying from equal slopes to sets of slopes with values as disparate as .1 and .9. The covariate population means were either equal or the smallest was .5 standard deviations less than the middle one and the largest was .5 standard deviations greater than the middle one. Simulation parameters were selected so that the study investigated situations in which the samples sizes, regression coefficients, and covariate means were combined in various ways with respect to size.

In each of the simulations, the proportion of F values exceeding the critical values for nominal alphas of .01, .05, and .10 were calculated. Comparisons of the obtained proportions to the corresponding nominal values were made with reference to appropriate confidence intervals.

Results

For those simulations conducted under the conditions of Case 1

(unequal error variances), the following conclusions seem appropriate:

1. When the treatment populations have equal covariate means, the F test of fixed-effects ANCOVA is remarkably robust to violations of the homogeneity of regression assumption as long as the treatment groups have equal sample sizes. There is a trend for the actual alpha values to exceed the corresponding values.
2. When the treatment populations have unequal covariate means, the F test is robust as long as the treatment group n's are equal. Significant differences between nominal and actual significance levels occur consistently only for extreme regression coefficients as disparate as .1 and .9. Again the trend is for the actual values to exceed nominal.
3. With unequal sample sizes the F test exhibits very little robustness regardless of whether the covariate means are equal or unequal. When small n's are paired with large regression coefficients, the actual alpha values tend to be less than the corresponding nominal values. When the small n is paired with the small coefficient the trend is reversed with actual values tending to be larger than nominal values.

When simulations are conducted under the conditions of Case 2 (equal error variances), the following conclusions appear appropri-

ate:

1. For either equal or unequal sample sizes when the treatment populations have equal covariate means, the F test of ANCOVA is robust with respect to even extreme violations of the homogeneity of regression assumption.
2. When the treatment populations have unequal covariate means and the sample sizes are equal, the F test of ANCOVA is robust with respect to violations of the homogeneity of regression assumption. Serious discrepancies between the nominal and actual significance levels occur consistently only for simulations in which the extreme regression coefficients are as disparate as .2 and .8. The trend under these conditions is for the actual alpha values to exceed the nominal values.
3. When the treatment populations have unequal covariate means and the sample sizes are unequal, the F test of ANCOVA is moderately robust with respect to violations of the homogeneity of regression assumption. Serious discrepancies between the nominal and actual significance levels occur regularly only for simulations in which the extreme regression coefficients are as different as .3 and .7. The trend again is for the actual alpha values to exceed the nominal values.

Contrary to the results stated by Peckham (1968) and McClaran (1972) the results of the present study indicate that the consequences of failing to satisfy the homogeneity of regres-

sion assumption in ANCOVA are more serious as sample size increases. Further investigation appears necessary to determine the cause of the discrepancy in results.

Comparisons of the results from Case 2 (unequal error variances) and Case 2 (equal error variances) provide evidence to indicate that, when both regression slopes and error variances are unequal, the lack of robustness in the ANCOVA F test is due primarily to the heterogeneity of error variances. Therefore, the present author suggest that experimenters be guided by the results of Case 2 (equal error variances) unless they have reason to suspect that they are using ANCOVA in a situation in which both regression slopes and variances are unequal.

REFERENCES

Dayton, C. M. The design of educational experiments. New York: McGraw-Hill, 1970

Feldt, L. S. A comparison of the precision of three experimental designs employing a concomitant variable. Psychometrika, 1958, 23, 335-353.

Glass, G. V., Peckham, P. D., & Sanders, J. R. Consequences of failure to meet assumptions underlying the fixed effects analysis of variance and covariance. Review of Educational Research, 1972, 42, 237-288.

Kempthorne, O. The design and analysis of experiments. New York: Wiley, 1952.

Lindquist, E. F. Design and analysis of experiments in psychology and education. Boston: Houghton Mifflin, 1953.

Marsagia, G. & Bray, T. A. A convenient method for generating normal variables. SIAM Review, 1964, 6-3, 260-264.

McClaran, V. R. An investigation of the effect of violating the assumption of homogeneity of regression slopes in the analysis of covariance model upon the F-statistic. (Doctoral dissertation, North Texas State University) Ann Arbor, Michigan: University Microfilms, 1972, No. 73-2919.

Peckham, P. D. An investigation of the effects of non-homogeneity of regression slopes upon the F-test of analysis of covariance. Boulder, Colorado: University of Colorado, Laboratory of Educational Research, Report No. 16, 1968.

Robinson, J. J. The effect of non-homogeneous within-group regression coefficients and sample size on the distribution of the F statistic in the analysis of covariance. (Doctoral dissertation, University of Oregon) Ann Arbor, Michigan: University Microfilms, 1969, No. 70-9468.

Winer, B.J. Statistical principles in experimental design (2nd ed.). New York: McGraw-Hill, 1971.

TABLE 1

Summary of Part I Simulations

						Case 1			Case 2		
						Actual α Values for Nominal of			Actual α Values for Nominal of		
n_1	n_2	n_3	β_1	β_2	β_3	.01	.05	.10	.01	.05	.10
3	5	7	.9	.5	.1	0.0061	0.0333*	0.0689*	0.0100	0.0517	0.1128
3	5	7	.8	.5	.2	0.0100	0.0422	0.0339*	0.0150*	0.0567*	0.1128
3	5	7	.7	.5	.3	0.0078	0.0444*	0.0794*	0.0078	0.0489	0.0933
3	5	7	.6	.5	.4	0.0089	0.0489	0.0994	0.0089	0.0494	0.0994
3	5	7	.5	.5	.5	0.0111	0.0533	0.0983	0.0094	0.0422	0.0917
3	5	7	.4	.5	.6	0.0106	0.0533	0.1039	0.0117	0.0489	0.0961
3	5	7	.3	.5	.7	0.0111	0.0700*	0.1289*	0.0100	0.0406	0.0894
3	5	7	.2	.5	.8	0.0228*	0.0878*	0.1500*	0.0117	0.0450	0.0972
3	5	7	.1	.5	.9	0.0350*	0.1078*	0.1872*	0.0150*	0.0578	0.1111
5	5	5	.5	.5	.5	0.0128	0.0517	0.0983	0.0111	0.0456	0.0989
5	5	5	.4	.5	.6	0.0117	0.0489	0.1017	0.0089	0.0444	0.0933
5	5	5	.3	.5	.7	0.0122	0.0489	0.0950	0.0139	0.0522	0.0978
5	5	5	.2	.5	.8	0.0094	0.0506	0.0956	0.0117	0.0550	0.1050
5	5	5	.1	.5	.9	0.0172*	0.0639*	0.1144*	0.0133	0.0506	0.1072
5	10	15	.9	.5	.1	0.0067	0.0372*	0.0744*	0.0139	0.0667*	0.1133
5	10	15	.8	.5	.2	0.0094	0.0344*	0.0733*	0.0117	0.0550	0.1178*
5	10	15	.7	.5	.3	0.0072	0.0306*	0.0711*	0.0150*	0.0617*	0.1167*
5	10	15	.6	.5	.4	0.0083	0.0344*	0.0828*	0.0100	0.0550	0.1044
5	10	15	.5	.5	.5	0.0106	0.0539	0.1022	0.0083	0.0494	0.0956
5	10	15	.4	.5	.6	0.0161*	0.0639*	0.1206*	0.0078	0.0544	0.0967
5	10	15	.3	.5	.7	0.0206*	0.0861*	0.1356*	0.0106	0.0439	0.0911
5	10	15	.2	.5	.8	0.0306*	0.1006*	0.1672*	0.0100	0.0600*	0.1050
5	10	15	.1	.5	.9	0.0494*	0.1183*	0.1883*	0.0094	0.0483	0.1039

TABLE 1 (continued)

n_1	n_2	n_3	β_1	β_2	β_3	Case 1 Actual α Values for Nominal of .01	Case 1 Actual α Values for Nominal of .05	Case 1 Actual α Values for Nominal of .10	Case 2 Actual α Values for Nominal of .01	Case 2 Actual α Values for Nominal of .05	Case 2 Actual α Values for Nominal of .10
10	10	10	.5	.5	.5	0.0083	0.0422	0.0917	0.0094	0.0528	0.1022
10	10	10	.4	.5	.6	0.0106	0.0544	0.1022	0.0072	0.0467	0.0956
10	10	10	.3	.5	.7	0.0106	0.0506	0.1056	0.0128	0.0450	0.1006
10	10	10	.2	.5	.8	0.0100	0.0511	0.1044	0.0111	0.0461	0.0989
10	10	10	.1	.5	.9	0.0139	0.0567	0.1050	0.0067	0.0417	0.0844*
10	20	30	.9	.5	.1	0.0061	0.0317*	0.0656*	0.0167*	0.0522	0.1022
10	20	30	.8	.5	.2	0.0044*	0.0361*	0.0800*	0.0100	0.0550	0.1094
10	20	30	.7	.5	.3	0.0056	0.0361*	0.0756*	0.0078	0.0461	0.1000
10	20	30	.6	.5	.4	0.0061	0.0400	0.0794*	0.0106	0.0550	0.1033
10	20	30	.5	.5	.5	0.0106	0.0483	0.0961*	0.0100	0.0506	0.1006
10	20	30	.4	.5	.6	0.0122	0.0644*	0.1128	0.0083	0.0494	0.1089
10	20	30	.3	.5	.7	0.0144	0.0850*	0.1572*	0.0083	0.0444	0.0983
10	20	30	.2	.5	.8	0.0317*	0.1056*	0.1783*	0.0061	0.0422	0.0989
10	20	30	.1	.5	.9	0.0400*	0.1139*	0.1883*	0.0117	0.0611*	0.1139
20	20	20	.5	.5	.5	0.0111	0.0506	0.0944	0.0128	0.0483	0.0956
20	20	20	.4	.5	.6	0.0117	0.0528	0.1094	0.0128	0.0461	0.0939
20	20	20	.3	.5	.7	0.0133	0.0483	0.1083	0.0122	0.0517	0.1061
20	20	20	.2	.5	.8	0.0089	0.0544	0.0978	0.0072	0.0444	0.1006
20	20	20	.1	.5	.9	0.0117	0.0600*	0.1078	0.0089	0.0511	0.1028

*Significantly different from nominal at the .95 level

TABLE 2

Summary of Part II Simulations

n_1	n_2	n_3	β_1	β_2	β_3	μ_1	μ_2	μ_3	Case 1 Actual α Values for Nominal of .01	.05	.10	Case 2 Actual α Values for Nominal of .01	.05	.10
3	5	7	.9	.5	.1	-.5	.0	.5	0.0083	0.0400	0.0728*	0.0167*	0.0678*	0.1222*
3	5	7	.8	.5	.2	-.5	.0	.5	0.0100	0.0417	0.0823*	0.0106	0.0617*	0.1178*
3	5	7	.7	.5	.3	-.5	.0	.5	0.0067	0.0406	0.0806*	0.0100	0.0467	0.0983
3	5	7	.6	.5	.4	-.5	.0	.5	0.0083	0.0439	0.0939	0.0111	0.0506	0.1017
3	5	7	.5	.5	.5	-.5	.0	.5	0.0056	0.0522	0.0994	0.0100	0.0533	0.1033
3	5	7	.4	.5	.6	-.5	.0	.5	0.0151*	0.0567	0.1044	0.0072	0.0544	0.1089
3	5	7	.3	.5	.7	-.5	.0	.5	0.0133	0.0656*	0.1211*	0.0100	0.0500	0.0961
3	5	7	.2	.5	.8	-.5	.0	.5	0.0333*	0.1011*	0.1511*	0.0156*	0.0611*	0.1150*
3	5	7	.1	.5	.9	-.5	.0	.5	0.0461*	0.1322*	0.2033*	0.0161*	0.0583	0.1239*
3	5	7	.9	.5	.1	.5	.0	-.5	0.0056	0.0417	0.0822*	0.0117	0.0650*	0.1233*
3	5	7	.8	.5	.2	.5	.0	-.5	0.0100	0.0467	0.0856*	0.0133	0.0561	0.1083
3	5	7	.7	.5	.3	.5	.0	-.5	0.0044*	0.0444	0.0800*	0.0106	0.0583	0.1183*
3	5	7	.6	.5	.4	.5	.0	-.5	0.0061	0.0472	0.0956	0.0122	0.0500	0.1017*
3	5	7	.5	.5	.5	.5	.0	-.5	0.0150*	0.0489	0.1006	0.0089	0.0494	0.0967
3	5	7	.4	.5	.6	.5	.0	-.5	0.0150*	0.0589	0.1144	0.0111	0.0511	0.0928
3	5	7	.3	.5	.7	.5	.0	-.5	0.0122	0.0678*	0.1356*	0.0117	0.0567	0.1206*
3	5	7	.2	.5	.8	.5	.0	-.5	0.0256*	0.0856	0.1339*	0.0106	0.0550	0.1139
3	5	7	.1	.5	.9	.5	.0	-.5	0.0461*	0.1339*	0.2083*	0.0167*	0.0628*	0.1211*
5	5	5	.5	.5	.5	-.5	.0	.5	0.0139	0.0522	0.1133	0.0094	0.0500	0.0994
5	5	5	.4	.5	.6	-.5	.0	.5	0.0133	0.0539	0.1094	0.0139	0.0550	0.1056
5	5	5	.3	.5	.7	-.5	.0	.5	0.0117	0.0483	0.1000	0.0078	0.0567	0.1061
5	5	5	.2	.5	.8	-.5	.0	.5	0.0178*	0.0706*	0.1239*	0.0067	0.0561	0.1206*
5	5	5	.1	.5	.9	-.5	.0	.5	0.0161*	0.0750*	0.1261*	0.0128	0.0617*	0.1183*

TABLE 2 (continued)

n_1	n_2	n_3	β_1	β_2	β_3	μ_1	μ_2	μ_3	Case 1 Actual α Values for Nominal of .01	.05	.10	Case 2 Actual α Values for Nominal of .01	.05	.10
5	5	5	.4	.5	.6	.5	.0	-.5	0.0161*	0.0539	0.0906	0.0033*	0.0450	0.0878
5	5	5	.3	.5	.7	.5	.0	-.5	0.0100	0.0506	0.1061	0.0100	0.0489	0.0994
5	5	5	.2	.5	.8	.5	.0	-.5	0.0122	0.0539	0.1028	0.0122	0.0567	0.1004
5	5	5	.1	.5	.9	.5	.0	-.5	0.0150*	0.0694*	0.1250*	0.0128	0.0566	0.1106
5	10	15	.9	.5	.1	-.5	.0	-.5	0.0111	0.0556	0.0994	0.0200*	0.0800*	0.1500*
5	10	15	.8	.5	.2	-.5	.0	-.5	0.0061	0.0400	0.0850*	0.0156*	0.0717*	0.1283*
5	10	15	.7	.5	.3	-.5	.0	-.5	0.0083	0.0467	0.0978	0.0117	0.0550	0.1156*
5	10	15	.6	.5	.4	-.5	.0	-.5	0.0078	0.0428	0.0900	0.0083	0.0544	0.1072
5	10	15	.5	.5	.5	-.5	.0	-.5	0.0100	0.0478	0.0928	0.0100	0.0517	0.1094
5	10	15	.4	.5	.6	-.5	.0	-.5	0.0172*	0.0650*	0.1111	0.0139	0.0500	0.1044
5	10	15	.3	.5	.7	-.5	.0	-.5	0.0217*	0.0794*	0.1522*	0.0139	0.0544	0.1050
5	10	15	.2	.5	.8	-.5	.0	-.5	0.0361*	0.1206*	0.1944	0.0172*	0.0661*	0.1200*
5	10	15	.1	.5	.9	-.5	.0	-.5	0.0739*	0.1822*	0.2606*	0.0211*	0.0800*	0.1433*
5	10	15	.9	.5	.1	.5	.0	-.5	0.0094	0.0467	0.0989	0.0194*	0.0839*	0.1606*
5	10	15	.8	.5	.2	.5	.0	-.5	0.0139	0.0478	0.0944	0.0144	0.0689*	0.1272*
5	10	15	.7	.5	.3	.5	.0	-.5	0.0089	0.0444	0.0844*	0.0178*	0.0667*	0.1300*
5	10	15	.6	.5	.4	.5	.0	-.5	0.0094	0.0383*	0.0744*	0.0122	0.0517	0.1011
5	10	15	.5	.5	.5	.5	.0	-.5	0.0111	0.0556	0.1033	0.0144	0.0517	0.0956
5	10	15	.4	.5	.6	.5	.0	-.5	0.0122	0.0644*	0.1156*	0.0050*	0.0478	0.1000
5	10	15	.3	.5	.7	.5	.0	-.5	0.0178*	0.0856*	0.1444*	0.0106	0.0561	0.1294*
5	10	15	.2	.5	.8	.5	.0	-.5	0.0400*	0.1167	0.1889*	0.0144	0.0706*	0.1200*
5	10	15	.1	.5	.9	.5	.0	-.5	0.0733*	0.1800*	0.2628*	0.0150*	0.0617*	0.1378*

TABLE 2 (Continued)

n_1	n_2	n_3	β_1	β_2	β_3	μ_1	μ_2	μ_3	Case 1 Actual α Values for Nominal of .01	.05	.10	Case 2 Actual α Values for Nominal of .01	.05	.10
10	10	10	.5	.5	.1	-.5	.0	.5	0.0128	0.0539	0.0983	0.0078	0.0472	0.1011
10	10	10	.4	.5	.2	-.5	.0	.5	0.0083	0.0483	0.1028	0.0089	0.0472	0.0989
10	10	10	.3	.5	.3	-.5	.0	.5	0.0089	0.0522	0.0928	0.0133	0.0628*	0.1128
10	10	10	.2	.5	.4	-.5	.0	.5	0.0150*	0.0706*	0.1133*	0.0094	0.0561	0.1106
10	10	10	.1	.5	.5	-.5	.0	.5	0.0189*	0.0711*	0.1211*	0.0156*	0.0744*	0.1222*
10	10	10	.4	.5	.1	.5	.0	-.5	0.0100	0.0489	0.0956	0.0111	0.0558	0.1061
10	10	10	.3	.5	.2	.5	.0	-.5	0.0094	0.0589	0.1089	0.0061	0.0506	0.1050
10	10	10	.2	.5	.3	.5	.0	-.5	0.0194*	0.0767*	0.1406*	0.0128	0.0606*	0.1128
10	10	10	.1	.5	.4	.5	.0	-.5	0.0200*	0.0800*	0.1539*	0.0150*	0.0700*	0.1306*
10	20	30	.9	.5	.1	-.5	.0	.5	0.0156*	0.0867*	0.1411*	0.0344*	0.1089*	0.1411
10	20	30	.8	.5	.2	-.5	.0	.5	0.0111	0.0622*	0.1233*	0.0261*	0.0994*	0.1233
10	20	30	.7	.5	.3	-.5	.0	.5	0.0078	0.0444	0.0994	0.0133	0.0611*	0.0994
10	20	30	.6	.5	.4	-.5	.0	.5	0.0094	0.0467	0.1000	0.0111	0.0494	0.1000
10	20	30	.5	.5	.5	-.5	.0	.5	0.0106	0.0533	0.1044	0.0094	0.0411	0.1044
10	20	30	.4	.5	.6	-.5	.0	.5	0.0072	0.0550	0.1156*	0.0083	0.0494	0.1156*
10	20	30	.3	.5	.7	-.5	.0	.5	0.0333*	0.0956*	0.1667*	0.0150*	0.0606*	0.1667*
10	20	30	.2	.5	.8	-.5	.0	.5	0.0556*	0.1511*	0.2322*	0.0250*	0.0994*	0.2322*
10	20	30	.1	.5	.9	-.5	.0	.5	0.0983*	0.2150*	0.3178*	0.0361*	0.1144*	0.3178

TABLE 2 (continued)

									Case 1 Actual α Values for Nominal of			Case 2 Actual α Values for Nominal of		
n_1	n_2	n_3	β_1	β_2	β_3	μ_1	μ_2	μ_3	.01	.05	.10	.01	.05	.10
10	20	30	.9	.5	.1	-.5	.0	.5	0.0128	0.0767*	0.1522*	0.0322*	0.1044*	0.1822*
10	20	30	.8	.5	.2	-.5	.0	.5	0.0111	0.0567	0.1117	0.0206*	0.0883*	0.1500*
10	20	30	.7	.5	.3	-.5	.0	.5	0.0094	0.0522	0.0989	0.0100	0.0539	0.1106
10	20	30	.6	.5	.4	-.5	.0	.5	0.0106	0.0467	0.0950	0.0144	0.0522	0.1006
10	20	30	.5	.5	.5	-.5	.0	.5	0.0111	0.0500	0.0939	0.0089	0.0483	0.1022
10	20	30	.4	.5	.6	-.5	.0	.5	0.0067	0.0606*	0.1189*	0.0139	0.0583	0.1211*
10	20	30	.3	.5	.7	-.5	.0	.5	0.0261*	0.0922*	0.1611*	0.0106	0.0589	0.1156*
10	20	30	.2	.5	.8	-.5	.0	.5	0.0606*	0.1550*	0.2372*	0.0211*	0.0922*	0.1611*
10	20	30	.1	.5	.9	-.5	.0	.5	0.0928*	0.2094*	0.3033*	0.0332*	0.1028*	0.1761*
20	20	20	.5	.5	.5	.5	.0	-.5	0.0111	0.0444	0.0928	0.0094	0.0467	0.1028
20	20	20	.4	.5	.6	.5	.0	-.5	0.0133	0.0483	0.1050	0.0128	0.0617*	0.1150*
20	20	20	.3	.5	.7	.5	.0	-.5	0.0089	0.0672*	0.1206*	0.0128	0.0583	0.1022
20	20	20	.2	.5	.8	.5	.0	-.5	0.0206*	0.0744*	0.1417*	0.0133	0.0728*	0.1256*
20	20	20	.1	.5	.9	.5	.0	-.5	0.0278*	0.1111*	0.1950*	0.0211*	0.0839*	0.1617*
20	20	20	.4	.5	.6	-.5	.0	.5	0.0117	0.0539	0.1000	0.0117	0.0517	0.0978
20	20	20	.3	.5	.7	-.5	.0	.5	0.0156*	0.0733*	0.1283*	0.0106	0.0528	0.1111
20	20	20	.2	.5	.8	-.5	.0	.5	0.0278*	0.0900*	0.1528*	0.0178*	0.0739*	0.1306*
20	20	20	.1	.5	.9	-.5	.0	.5	0.0344*	0.1006*	0.1744*	0.0167*	0.0817*	0.1650*

*Significantly different from nominal at the .95 level